

NCMAS 2020 Proposal

Title: Star Formation and Feedback in a Turbulent Interstellar Medium

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Project: jh2

1. Project aims and outcomes

The overarching goal of this project is to understand the formation of stars and their feedback on the interstellar medium (ISM). Because the problem as a whole is too daunting to allow direct simulation of the full system, we identify individual sub-parts of the problem that we can simulate with reasonable fidelity. For 2020 we plan work on four sub-projects.

1.1. Radiatively-driven galactic winds (supports DP190101258)

Background. One of the most dramatic forms of stellar feedback is galactic winds: flows of mass out of the star-forming discs of galaxies into the circumgalactic medium. While it has long been known that supernovae can drive winds, there remains significant uncertainty about whether the pressure exerted by starlight might also contribute, or even dominate, wind production under some circumstances. The uncertainty arises because the rate at which photons are able to transfer momentum to gas is limited by the radiation Rayleigh-Taylor instability (RRTI; [Jacquet & Krumholz 2011](#), [Figure 1](#)), which is analogous to the usual fluid Rayleigh-Taylor instability but with the light fluid replaced by a radiation field. RRTI allows gas to escape through optically thin channels, greatly reducing the momentum delivered to gas, and whether radiation can drive galactic winds depends on the rate

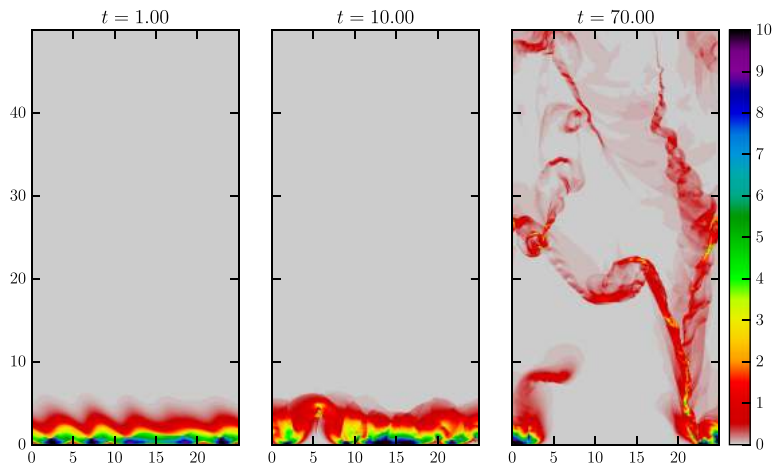


Fig. 1.— Snapshots at three dimensionless times in simulations of RRTI ([Wibking et al. 2018](#)). In the simulation box shown, gravity points downward and an isotropic radiation flux enters the computational domain from the bottom, interacting with the gas therein; boundary conditions are closed at the bottom, open at the top, and periodic horizontally. Colours show gas density in dimensionless units. In this simulation the radiation field amplifies small initial perturbations in the density (left panel) which grow to non-linear scales (middle panel) until much of the gas is catastrophically ejected (right panel).

of radiation-gas momentum transfer once the instability saturates. [Krumholz & Thompson \(2012, 2013\)](#) were the first to carry out radiation-hydrodynamic simulations of RRTI, but many other authors have followed (e.g., [Davis et al. 2014](#); [Rosdahl & Teyssier 2015](#); [Tsang & Milosavljević 2015](#); [Wibking et al. 2018](#)). The preliminary indication of the numerical experiments is that radiation fields can drive winds, but that only a tiny fraction of galaxies are luminous enough to do so; radiation is unimportant for most galaxies ([Crocker et al. 2018a,b](#)). However, the numerical experiments have a major limitation: none have simultaneously treated both the highly-beamed direct radiation field that emerges from stars and the much more diffuse field that results when interstellar dust reprocesses the starlight into the infrared. However, models suggest that the regime where winds are most likely to be launched is when both types of radiation field contribute about equally.

Proposed work. We propose to carry out numerical experiments of the non-linear saturation of RRTI, using the [Krumholz & Thompson \(2012\)](#) setup adopted by most subsequent authors, and illustrated in [Figure 1](#): a box of gas with a uniform downward gravitational field into which radiation is injected from below. We vary the strength of the radiation field and the optical depth of the gas column, allow the instability to saturate, then measure the rate of mass ejection from the box and the rate of momentum transfer from radiation to gas. For the first time we will include both direct and diffuse radiation, by combining the long characteristics method of [Rosen et al. \(2017\)](#) with a novel GPU-accelerated M1 scheme ([Ripoll et al. 2001](#); [González et al. 2007](#)) that we are developing to handle the diffuse radiation field; this should be significantly more accurate than the flux-limited diffusion method used in the [original Krumholz & Thompson \(2012\) work](#). In 2020 we a plan to complete development of the M1 scheme and begin a campaign of simulations the explore the vital overlap regime, which will continue into 2021.

Outcome and significance. Our simulations will produce the first exploration of RRTI including both direct and diffuse radiation, the regime of greatest astrophysical interest. In this regime, we will measure the radiation-gas momentum transfer rate as a function of the system parameters, identifying under what circumstances radiation can launch winds. Using the methods developed by [Crocker et al. \(2018a,b\)](#), we can in turn map these dimensionless parameters to observed galaxy properties, thereby settling the question of whether radiation pressure ever contributes to galactic winds. This will resolve a significant unsolved problem in galaxy formation theory.

1.2. Galactic chemodynamics [\(supports FT180100375\)](#)

Background. Project two focuses on another important aspect of star formation feedback: the enrichment of interstellar gas with heavy elements, and the subsequent incorporation of these elements into a next generation of stars. This is a topic of great current interest in astronomy because modern highly-multiplexed spectrographs and integral field units (IFUs) have made it possible in observing campaigns of a few years to obtain spectra, and thereby measure chemical element abundances, in millions of Milky Way stars, and across the interstellar media of thousands of nearby Milky Way-like galaxies (e.g., [De Silva et al. 2015](#); [Allen et al. 2015](#)). In principle this abundance information encodes a tremendous amount of information about galaxy assembly, since heavy elements act like “tracer dyes” in the ISM. However, to interpret the meanings of these tracer dyes we must understand how they flow through interstellar space and are subsequently incorporated into stars. [Our group has been leading efforts to answer these questions, both through numerical simulations \(e.g., Yang & Krumholz 2012; Feng & Krumholz 2014; Petit et al. 2015; Fujimoto et al. 2018; Armillotta et al. 2018\) and pencil-and-paper analysis \(Krumholz & Ting 2018\).](#)

Proposed work. We propose to run simulations of ISM elemental enrichment for two sets of galaxy parameters: one Milky Way-like, and one M83-like. The simulations run in two stages. The first is simulations of isolated Milky Way-like galaxies at a resolution of ~ 5 pc, sufficient to resolve individual supernova explosions and the elements they inject. We simulate until the elemental distribution reaches statistical equilibrium ([Fujimoto et al. 2018](#)); we show typical steady-state galaxies in [Figure 2](#). [Our first generation of simulations](#) included only elements produced by type II supernovae, but we have recently upgraded our code to include elements synthesised in asymptotic giant branch (AGB) stars and type Ia supernovae. We will also upgrade our existing simulations by introducing an improved model for pre-supernova stellar feedback, which detailed tests of [our](#) first generation model proved are required to reproduce the observed correlation between gas and newborn stars on < 100 pc scales ([Fujimoto et al.](#)

2019). The second stage follows the approach developed in [Armillotta et al. \(2018\)](#): we extract ~ 100 pc regions from the galactic simulation that are on the verge of collapse, and re-simulate them at $\sim 10^{-3}$ pc resolution, sufficient to capture the formation of individual stars. In addition to the improved initial conditions provided by our second generation galaxy simulations, we will improve on the first-generation zoom-in simulations by including stellar feedback and magnetic fields, which we omitted in the first generation runs.

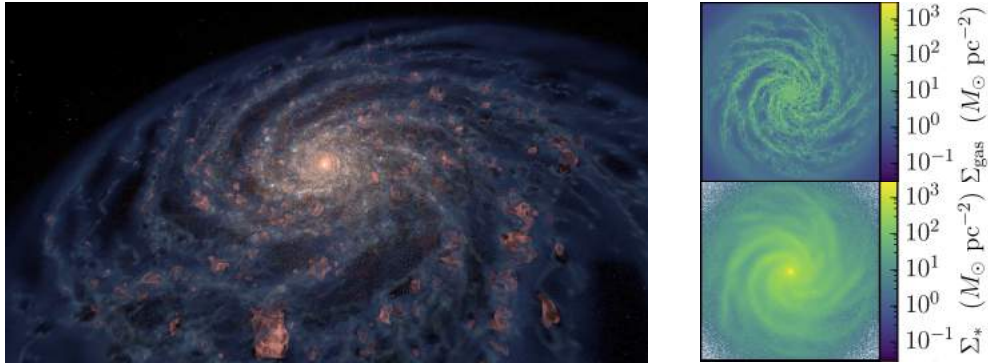


Fig. 2.— Results from a simulation of a Milky Way-like galaxy ([Goldbaum et al. 2016](#)). Left: a volume-rendering of the gas and stars. Right: gas (top) and stellar (bottom) surface densities in a $20 \text{ kpc} \times 20 \text{ kpc}$ projection. The images show the steady-state distributions obtained after 600 Myr of evolution.

Outcome and significance. Our simulations will have two primary outcomes. At the galactic scale, we will for the first time be able to predict the distributions of multiple elements, and explore how those distributions vary as a function of elements’ astrophysical origin sites and of the physical properties of the galaxy. These simulations will be directly comparable to the observed abundance maps coming out of IFU surveys, and will be useful in testing propositions based on those observations, for example that galaxy metallicity gradients are diagnostic of the strength of galactic winds (e.g., [Ho et al. 2015](#)). At the stellar scale, we will determine the relationship between chemical and physical space over scales of $\sim 10 - 100$ pc and times of ~ 10 Myr. This is precisely what is needed to interpret observations emerging from Australian stellar abundance surveys such as GALAH ([De Silva et al. 2015](#)), i.e., we will be able to say whether a measured 10% difference in abundances across elements with different astrophysical origin sites indicates that the stars were born within ~ 1 pc of one another, within ~ 10 pc, or within ~ 1 kpc.

1.3. The life cycle of galaxy centres (supports [DP190101258](#) and [UA-DAAD](#))

Background. Our third project focuses on star formation at the centres of nearby galaxies, including our own. These regions are particularly important because they are the closest local analogs to the dense, warm conditions under which stars formed in the early Universe (e.g., [Kruijssen & Longmore 2013](#)). Indeed, there is strong observational evidence that Milky Way’s central molecular zone (CMZ) undergoes periodic cycles of starburst and quenching (e.g., [Kruijssen et al. 2014](#); [Krumholz & Kruijssen 2015](#); [Krumholz et al. 2017](#)), and that it drives a wind of neutral hydrogen out of the Galaxy (e.g., [McClure-Griffiths et al. 2013](#); [Di Teodoro et al. 2018](#)); both these features are expected to be ubiquitous in high-redshift galaxies. In the last year [we carried out a first simulation of the Milky Way CMZ](#) ([Armillotta et al. 2019](#), see Progress Report for further details), and showed that we indeed produce a wind, and reproduce the observed burst/quench cycle.

Proposed work. In the coming year we will carry out two additional studies. First, since the exact gravitational potential of the stars in the Milky Way CMZ is only poorly known, we will **repeat** the [Armillotta et al. \(2019\)](#) simulation with an alternative version of the potential, so as to determine the sensitivity of CMZ dynamics to the potential. Second, we wish to carry out an analogous study of the centre of the nearby galaxy M83, using the same physics but with a gravitational potential and gas content chosen to mimic that galaxy rather than the Milky Way. M83 is an attractive target because it is relatively similar to the Milky Way in bulk properties, but appears to be at the opposite end of its burst/quench cycle (M83 is near maximum star formation rate, Milky Way near minimum), allowing comparison to observations when in a very different state. Moreover, M83 has recently been the subject of an extensive survey with the Atacama Large Millimetre Telescope (ALMA; Callanan et al., in prep.), so excellent comparison data are available, **to which the CI of this proposal has access via his membership in the collaboration that obtained the data.**

Outcome and significance. We anticipate two main scientific outcomes of this study. First, we will determine the sensitivity of the Milky Way CMZ’s star formation and wind cycle to the stellar potential of the Galaxy. Not only will this help us understand our own Galaxy better, such a study will reveal to what extent the phenomenology of the Milky Way centre – for example the burst period – is ubiquitous, and to what extent it depends on peculiarities of our Galaxy. Second, we will learn if M83 and the Milky Way really are similar in most respects, and most importantly if M83 is expected to have a wind similar to the Milky Way’s. Answers to these questions will be crucial for interpreting the ALMA data.

1.4. Stochasticity-robust stellar population inference **(supports FT180100375)**

Background. The first three projects focus on direct simulation of star formation and the ISM; this fourth project instead focuses on novel methods for interpreting astronomical observations that are relevant to the first three projects. Beyond the Milky Way and its closest neighbours, we generally cannot resolve individual stars; instead, we infer properties of the observed system from the integrated light of all the stellar population, plus the nebular line emission produced when starlight is processed through the ISM. The nebular light is critical to measurements of elemental abundances (e.g., [Kewley et al. 2019](#)). As IFUs have allowed us to resolve smaller and smaller regions of galaxies, however, we have encountered the problem of stochasticity. Stellar masses are randomly drawn from a distribution known as the initial mass function (IMF), and if one observes an entire galaxy, one can reasonably assume that the observed light will be an average over the IMF. However, that average tends to be dominated by a few very rare, massive stars, which are far brighter than their more numerous low-mass cousins. In the small regions that IFUs are now probing, the expected number of such massive stars may be $\lesssim 1$, leading to large stochastic fluctuations in light output; two stellar populations whose bulk properties (e.g., total mass and age) are nearly identical may nevertheless have very different luminosity and colour because one happens to contain a very massive star and the other does not. In the stochastic regime, traditional statistical methods (for example χ^2 minimisation) for determining the properties of the stellar population producing the light fail, because the mapping from physical properties (mass, age) to light is non-deterministic.

Proposed work. **Our group has developed the Stochastically Lighting Up Galaxies (SLUG) code to perform inferences in this regime (da Silva et al. 2012, 2014; Krumholz et al. 2015a), and we have demonstrated successful that SLUG can produce reliable posterior probability distribution functions (PDFs), properly accounting for stochastic fluctuations, on both observed galaxies (Krumholz et al. 2015b, Figure 3) and synthetic data (Krumholz et al.**

2019). The basic technique is explained below, in [Section 2.4](#), but a short summary is that one uses a library of Monte Carlo realisations of the stellar population, and then compares observed systems to the library. Thus far [our](#) work with SLUG has focused on recovering the properties of star clusters, but we now wish to extend that technique to the problem of recovering elemental abundances as revealed by the nebular lines produced when the light of young clusters is reprocessed by the ISM. This is computationally-challenging problem because of the large number of Monte Carlo realisations required, and the complexity of calculations of nebular reprocessing.

Outcome and significance. The primary outcome of calculation will be the library of Monte Carlo simulations of line emission from interstellar gas, which we will use to build a tool for inferring elemental abundances from emission lines, properly accounting for stochastic fluctuations in stellar populations. As with the rest of the SLUG package, this tool will be freely available, and will not require supercomputing resources to use; constructing the initial sample library is computationally expensive, but the parameter estimation step can be done on a workstation. Based on current usage of other SLUG-based tools, we anticipate wide adoption of the method by researchers carrying out high resolution mapping of metal abundances, for example large Australian IFU surveys such as SAMI ([Allen et al. 2015](#)). An additional benefit of this research is that we will determine which emission lines are most robust against stochastic fluctuations, which will be helpful in planning future surveys.

2. Numerical methods

[Our group](#) uses a number of different codes, some developed in-house ([ORION](#) and [SLUG](#)) and some public. Project 1 will use the [ORION](#) code, which is specialised for radiation-magnetohydrodynamics (MHD) and star formation. Project 2 will use [ORION](#) and [enzo](#), another code that is specialised for galaxies and cosmology. Project 3 will use [GIZMO](#), a meshless hydrodynamics code that has advantages for simulating the high bulk velocity flows found in galactic centres. Finally, project 4 will use the [SLUG](#) code discussed above. Here we summarise the physical models and algorithms used in each code, leaving a detailed discussion of workflows to the Resources section.

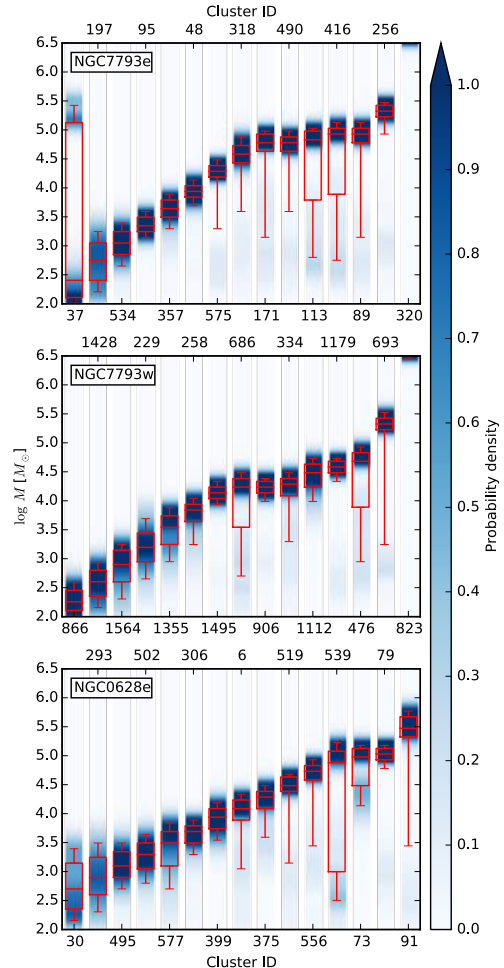


Fig. 3.— Example application of SLUG ([Krumholz et al. 2015b](#)). Each stripe indicates the marginal posterior probability distribution for mass inferred for a star cluster in the *Hubble Space Telescope* LEGUS survey ([Calzetti et al. 2015](#)); clusters are labelled by catalog ID number. Red boxes indicate the 2nd and 3rd quartiles, and whiskers mark the 10th to 90th percentile range. Stochasticity causes some clusters to have extraordinarily broader posteriors.

2.1. ORION

ORION is a block-structured adaptive mesh refinement (AMR) code based on the Berger & Olinger (1984) and Berger & Colella (1989) method. The simulation begins on a uniform Eulerian grid, but as it runs the code dynamically adds finer Eulerian grids to cover regions of interest, based on user-specified criteria – in our case, regions that are self-gravitating (Truelove et al. 1997) or where there are sharp gradients in the radiation field indicating fronts subject to RRTI (e.g. Rosen et al. 2016). Within this framework there are three second-order accurate physics modules, which operate sequentially in an operator-split manner. The first solves the equations of ideal, compressible MHD using an unsplit, TVD, Godunov, constrained-transport scheme that maintains zero magnetic divergence to machine precision (Li et al. 2012). The second solves the Poisson equation of self-gravity using a multigrid method (Martin et al. 2008). The third solves the equation of radiative transfer using the hybrid adaptive ray-moment method developed by Rosen et al. (2017), which combines long characteristics to follow the highly-beamed radiation field from point sources with flux-limited diffusion (FLD) to follow the radiation field reprocessed by dust (Krumholz et al. 2007). We plan to replace the FLD method with an M1 method, accelerated by GPU, during 2020; the M1 method should be significantly more accurate, and, thanks to GPU acceleration, may prove to be faster as well. In addition to the physics modules, the code can represent stars by using accreting, radiating sink particles (Krumholz et al. 2004) coupled to a subgrid stellar evolution model (Offner et al. 2009). We will use this capability for project 2, where we wish to follow the formation of individual stars which must then be able to feed back on the gas around them with radiation and outflows.

2.2. enzo

enzo is also an AMR code, using the same basic architecture as ORION. It solves the Euler equations for compressible gas dynamics plus the Poisson equation for gravity (Bryan et al. 2014). The gas equations are solved simultaneously with the equations describing the evolution of a large ensemble of collisionless point masses that can represent either stars or dark matter. The hydrodynamic solver is a piecewise parabolic Godunov method, and enzo solves the Poisson equation for self-gravity using a particle-mesh Fourier transform method. The code also implements radiative heating and cooling via the GRACKLE package (Smith et al. 2017), which provides a set of pre-tabulated heating cooling rates as a function of density and temperature. We have modified the public version of enzo by adding a new sub-grid model for stochastic star formation and nucleosynthesis, based on SLUG, which we use to calculate stellar feedback and element yields. Compared to ORION, enzo’s advantage for galaxy-scale simulations is that it is much more efficient at handling the large numbers of collisionless particles required to represent old stars and dark matter.

2.3. GIZMO

GIZMO is a smoothed particle hydrodynamics-type arbitrary Lagrangian-Eulerian (ALE) code for MHD plus gravity. The MHD method uses a Riemann solver that can operate in either finite mass (Lagrangian) or finite volume (Eulerian) mode (Hopkins 2015; Hopkins & Raives 2016), while gravity uses a distributed-memory tree method descended from that in the popular cosmology code GADGET (Springel 2005). The scheme is second-order accurate in space and time. As with enzo, we have modified GIZMO to include stochastic stellar feedback based on SLUG. We have selected GIZMO rather than enzo for our CMZ simulations because in the CMZ there are very high streaming velocities ($\sim 200 \text{ km s}^{-1}$), but due to the organised nature of the flow the relative velocities between nearby fluid elements tend to be substantially smaller. In this configuration a Lagrangian method allows significantly larger time steps.

2.4. SLUG

SLUG is a stellar population synthesis (SPS) code, meaning that it combines pre-tabulated calculations of stellar evolution with models of stellar atmospheres, response functions for telescope filters, and tables of nucleosynthetic yields, to predict the light and nucleosynthetic yields for populations of stars with specified mass, age and metallicity distributions (da Silva et al. 2012, 2014; Krumholz et al. 2015a,b, 2019). The primary distinction between SLUG and other SPS codes is that, rather than simply using the input distributions to calculate mean values for the stellar output, SLUG carries out Monte Carlo calculations to determine the full probability distribution of outputs for a given input distribution. In addition to the basic Monte Carlo tool, the SLUG software suite includes a tool called `bayesphot` to solve the inverse problem: given the light output by an observed system, what should one infer about the physical properties (e.g., mass, age) of the stars producing the light? This tool operates on a large library of Monte Carlo simulations of stochastic light output produced by SLUG; we are requesting the CPU time required to build a library suitable for using `bayesphot` on emission line data. The basic approach is to use the library to build a Gaussian kernel density estimate (KDE) for the joint PDF of physical properties and light output. We then use KD-tree techniques operating in $N \ln N$ time to convolve this PDF with a Gaussian distribution representing the luminosity of an observed system in some filter, and the error distribution around it. We then project the convolved KDE into the physical dimensions to derive marginal posterior probability distributions for variables of interest.

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